Acadiana Sky

Finding the Mass of a Planet

Originally developed for the Lafayette Science Museum, Lafayette, LA

Astronomers learned long ago that it was possible to calculate the mass of a planet if there was an object going around it. The equation for doing this is

$$M_p = \frac{4\pi^2 d^3}{GT^2}$$

where M_p = the unknown mass of the planet,

π = 3.14,

D = the average distance between the centers of the planet and its satellite,

G = the Universal Gravitational Constant, and

T = the orbital period of the satellite in seconds.

The biggest drawback to this technique is that the orbiting body has to be far less massive than the planet. Happily, this is true for most of the planets and moons in our solar system.

With the beginning of the Space Age, it became possible to send artificial satellites to the planets and moons. By carefully tracking a satellite once it was in orbit around a body, it became possible to refine greatly our knowledge of planetary and lunar masses.

Find the masses of the following bodies. Use meters, kilograms, and seconds (the MKS System). The MKS value of the Universal Gravitational Constant (called "G") is 6.6743×10^{-11} meters^3 / kilogram-second^2.

Note to Teachers

The toughest part of this activity seems to be keeping the units straight. Mistakes of that kind generally make answers wrong by factors of a thousand or more. Other than that, the equation that is used should give answers within a few percent of accepted values (expect small errors due to rounding, and different ways that calculators handle arithmetic functions). The accepted values used here are from the NASA Space Science Data Coordinated Archive (NSSDCA), accessed in 2021 at https://nssdc.gsfc.nasa.gov/planetary/factsheet/ .

All of these calculations will be *much* easier if scientific notation is used, but they can be done without that. Calculators are highly recommended. Round all numbers to 2 decimal places.

In the 6 questions below, needed distances and orbital periods are within each question, usually given in the commonly used units of kilometers and days. Students *will* need to convert them to meters and seconds (there are 3600 seconds in an hour and 86,400 seconds in a day). The masses—the correct answers— are as follows:

Planet or Moon	**Mass**
Mercury	3.30×10^{23} kg
Venus	4.87×10^{24} kg
Earth	5.97×10^{24} kg
Mars	6.42×10^{23} kg
Jupiter	1.90×10^{27} kg
Saturn	5.68×10^{26} kg
Uranus	8.68×10^{25} kg
Neptune	1.02×10^{26} kg
Earth's Moon	7.34×10^{22} kg

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Teacher Information

(A Student Activity Questions handout follows this information)

1) A space station is in a circular orbit at an altitude of 500 km above the Earth's surface, orbiting the planet every 1.58 hours. The radius of Earth is 6378 km. What is the mass of Earth?

$d = 6378\ km + 500\ km = 6878\ km = 6.88 \times 10^3\ km = 6.88 \times 10^6\ m$

$T = 1.58\ h = 5688\ sec = 5.69 \times 10^3\ sec$

$$M_p = \frac{4\pi^2 d^3}{GT^2} = \frac{4(3.14)^2(6.88x10^6)^3}{(6.67x10^{-11})(5.69x10^3)^2} = \frac{39.48(3.26x10^{20})}{(6.67x10^{-11})(3.21x10^7)} = \frac{1.29x10^{22}}{2.14x10^{-3}} = 6.03x10^{24}\ kg$$

Any answer near $6x10^{24}$ kg is pretty accurate.

2) In 1968, the Apollo 8 spacecraft orbited the moon at an altitude of 111 km, with an orbital period of 1.96 hours. The radius of the moon is 1738 km. Calculate the mass of the moon.

$d = 1738 + 111 = 1849\ km = 1.85 \times 10^3\ km = 1.85 \times 10^6\ m.$

$T = 1.96\ h = 7056\ seconds = 7.06 \times 10^3\ seconds.$

$$M_p = \frac{4\pi^2 d^3}{GT^2} = \frac{39.48(1.85x10^6)^3}{(6.67x10^{-11})(7.06x10^3)^2} = \frac{39.48(6.33x10^{18})}{3.32x10^{-3}} = 7.53x10^{22}\ kg$$

Any answer near $7.5x10^{22}$ kg is pretty accurate.

3) In 1976, the Viking 1 spacecraft was placed in a looping orbit around Mars to prepare for sending a lander to the surface. The average distance of Viking 1 from the center of Mars was 20,338 km, and the orbital period was 8.86×10^4 seconds. Find the mass of Mars. Then convert the Viking 1 orbital period into hours. How does it compare with the length of a Martian day of 24.6 hours?

$d = 20{,}338\ km = 2.03x10^4\ km = 2.03x10^7\ m.$ $T = 8.86x10^4\ seconds.$

$$M_p = \frac{4\pi^2 d^3}{GT^2} = \frac{39.48(2.03x10^7)^3}{(6.67x10^{-11})(8.86x10^4)^2} = \frac{39.48(8.37x10^{21})}{(6.67x10^{-11})(7.85x10^9)} = 6.30x10^{23}\ kg$$

Any answer a little bigger than 6 x 10^{23} kg is pretty accurate.

Divide the Viking one orbital period by 3600 seconds in an hour to get (8.86 x 10^4)/3600 = 24.61 hours, essentially identical to one Martian day (called a "sol"). This very elongated orbit had its low point each sol near the expected mission landing zone in order to get high resolution images. As the spacecraft climbed out away from Mars to its highest point, and then orbited back to the lowest point, Mars made a complete rotation that allowed imaging of other parts of the planet.

4) Jupiter's moon Europa orbits the planet at a distance of 671,100 kilometers with an orbital period of 3.55 days. Determine the mass of Jupiter.

d = 671,000 km = 6.71 x 10^5 km = 6.71 x 10^8 m.
T = 3.55 d = 3.07 x 10^5 seconds.

$$M_p = \frac{4\pi^2 d^3}{GT^2} = \frac{39.48(6.7x10^8)^3}{(6.67x10^{-11})(3.07x10^5)^2} = \frac{39.48(3.01x10^{26})}{(6.67x10^{-11})(9.42x10^{10})} = 1.89x10^{27}\ kg$$

Any answer near 1.9 x 10^{27} is good.

Divide Jupiter's mass by the mass of Earth determined in question 1. How much more massive is Jupiter than Earth?

Using actual values, the ratio is 1.90 x 10^{27} / 5.97 x 10^{24} = 318. Student answers should be in the neighborhood of 300. Jupiter is in fact more massive than the masses of all the other planets in our solar system combined! You can let the students discover this surprising fact for themselves by giving them the actual values of all the planetary masses and simply letting them add the up the masses of the planets other than Jupiter.

5) Saturn has over 80 known moons. Its biggest moon, Titan, is 5150 kilometers in diameter, and orbits Saturn at a distance of 1,221,865 kilometers with an orbital period of 15.95 Earth-days. What is the mass of Saturn?

d = 1,221,865 km = 1.22 x 10^6 *km = 1.22 x* 10^9 *m.*

T = 15.95 d = 1.38 x 10^6 *seconds.*

$$M_p = \frac{4\pi^2 d^3}{GT^2} = \frac{39.48(1.22x10^9)^3}{(6.67x10^{-11})(1.38x10^6)^2} = \frac{39.48(1.82x10^{27})}{(6.67x10^{-11})(1.90x10^{12})} = 5.66x10^{26}\ kg$$

Any value near that is acceptable.

6) Assume Saturn is a perfect sphere with a radius of 58,232 kilometers. Calculate its volume using the equation

$$V = \frac{4}{3}\pi r^3 .$$

r = 58,232 km = 5.82 x 10^4 *km = 5.82 x* 10^7 *m.*

$$V = \frac{4}{3}\pi r^3 = 4.19(5.82x10^7)^3 = 4.19(1.97x10^{23}) = 8.25x10^{23}\ m^3$$

Acceptable answers should be very close to this value.

Divide the mass of Saturn as determined in Question 5 by its volume to obtain its average density. How does that compare with the density of water (1000 kg/meter^3)? Given enough water, would Saturn float or sink?

Average density = M/V = 5.66 x 10^{26} kg / 8.25 x 10^{23} m^3 = 686 kg/m^3.

Since this is less than the density of water, Saturn would float! To end your lesson with some bad astronomical humor: If you had a big enough bathtub and put Saturn in the tub, it would float. However, you would never want to do that because…Saturn might leave a ring.

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Student Activity Questions

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3) In 1976, the Viking 1 spacecraft was placed in a looping orbit around Mars to prepare for sending a lander to the surface. The average distance of Viking 1 from the center of Mars was 20,338 km, and the orbital period was 8.856×10^4 seconds. Find the mass of Mars. Convert the Viking 1 orbital period into hours. How does it compare with the length of a Martian day of 24.6 hours?

4) Jupiter's moon Europa orbits the planet at a distance of 671,100 kilometers with an orbital period of 3.55 days. Determine the mass of Jupiter.

Divide Jupiter's mass by the mass of Earth determined in question 1. How much more massive is Jupiter than Earth?

5) Saturn has over 80 known moons. It's biggest moon, Titan, is 5150 kilometers in diameter, and orbits Saturn at a distance of 1,221,865 kilometers with an orbital period of 15.95 Earth-days. What is the mass of Saturn?

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$$V = \frac{4}{3}\pi r^3 .$$

Divide the mass of Saturn as determined in Question 5 by its volume to obtain its average density. How does that compare with the density of water (1000 kg/meter^3)? Given enough water, would Saturn float or sink?